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Technical Notes

Optimum Design of Turboprop Engines Using Genetic Algorithm

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Nomenclature

C	=	work output coefficient
Cp	=	specific heat
F	=	thrust
f	=	fuel-to-air ratio
k_1 to k_9	=	constants in offdesign equations
Lcv	=	fuel heating value
M	=	Mach number
MFP_9	=	mass flow parameter at nozzle exit
$\dot{m}$	=	mass flow
P	=	pressure
T	=	temperature
V	=	velocity
Ws	=	specific power
η	=	efficiency
π	=	pressure ratio
τ	=	temperature ratio

Subscripts

b	=	burner
C	=	compressor, core
g	=	gear
H	=	high pressure
L	=	low pressure
m	=	mechanical, mixing losses due to cooling
prop	=	propeller
r	=	ram
T	=	turbine
t, tot	=	total

I. Introduction

TURBOPROP engines came into wide use with a need for higher thrust levels and reduced fuel consumption at relatively low flight speeds. However, their development has to be reconsidered because they were quite modest. Higher compressor pressure ratios (CPRs) and turbine inlet temperatures (TITs) are still difficult to achieve, owing to small blades in the rear stages of the compressor and the turbine which do not allow using sophisticated cooling techniques. Morris [1] showed that if size effects were ignored, a CPR in excess of 30 and a TIT of 1700 K could be reached. Brooks and Hirschkron [2] arrived at a CPR of 17–20 and a TIT of 1535–1590 K, based on turboprops for commuter aircraft.

In constrained optimization problems, direct methods start with a single design point and use the local gradient of the objective function to determine a search direction. Such methods are efficient as long as the objective functions are differentiable and convex [3]. These limits have led to other heuristic optimization methods, particularly the genetic algorithms (GAs) [4], because they are capable of searching the entire solution space with an increased likelihood of finding the global optimum. These algorithms are very well suited to deal with multi-objective real design problems (MOPs) because they make use of an evolving population of solutions that is driven toward the set of the true tradeoff. Since the original nondominated sorting procedure given by Goldberg [4] (considered as the catalyst for several different versions of multi-objective optimization algorithms), there has been a growing interest in devising GAs for MOPs, such as the vector evaluated GA (VEGA) proposed by Schaffer [5] and the nondominated sorting GA (NSGA) by Srinivas and Deb [6].

The present paper describes an approach that gives the freedom to select or optimize the design of new turboprops to match the power requirements of a propeller-driven L100-30 aircraft powered by four turboprops, for which flight performance and a diagram of constraints were established. In such thermal systems, the Pareto solutions provide more insights into the competing objectives: the minimum power specific fuel consumption (PSFC) can be directly translated into increased range and payload, whereas, the high specific power leads to reducing engine size, weight, and installation losses. Three configurations of turboprops included a single-spool fixed turbine, a single-spool free turbine, and a twin-spool fixed turbine that were modeled within the developed engine performance analyzer. The obtained results based on the model of aircraft have indicated that, by adopting the GA PIKAIA for this two-objective optimization problem, we could preserve the diversity of nondominated individuals and the quality of the Pareto front, and the decision variables related to the propulsion cycle could be determined easily.

II. Flight Performance Analysis

Propeller-driven aircraft are designed to meet short takeoff and landing (STOL) requirements and performance during climb and cruising to conform to the Federal Air Regulation standards in FAR 25 [7]. For this studied aircraft, the optimum cruise is at Mach = 0.475 and an altitude of 28,000 ft (8535.4 m) [8], whereas the takeoff is at Mach = 0.1 at sea level on a hot day. Analysis of performance in different flight mission segments have led to the constraints diagram in terms of wing loading and power loading (not shown here), based on available drag polar [9]. The envelope function used to plot the constraints' boundaries [10] allowed the determination of the matching point close to maximum power loading and wing loading. By considering a value of wing loading of $4251.76\ \mathrm{N/m^2}$, the power loading had a value of $0.04823\ \mathrm{N/W}$, and subsequently had a takeoff power of 4795 hp per engine.

III. Propulsion Cycle Analysis

Three configurations of turboprop engines are considered: single-spool, free power turbine, and twin-spool fixed power turbine, shown with stations numbered in Fig. 1.

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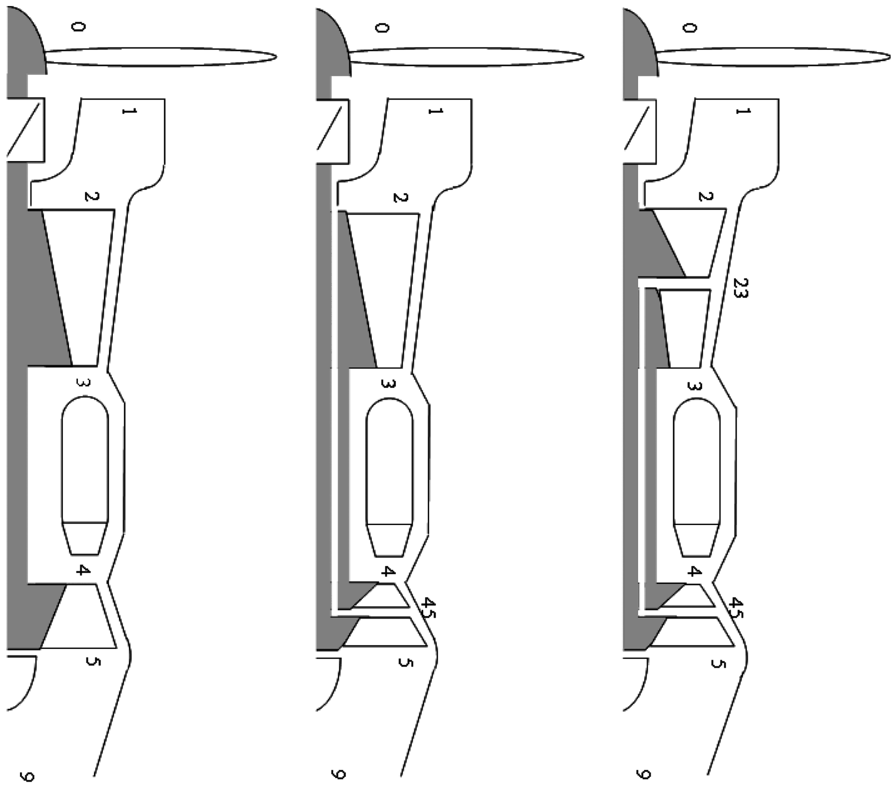


Fig. 1 Turboprop configurations and stations numbering.

A. On-Design Performance

It is more appropriate to consider the work supplied to the propeller rather than the thrust by means of a dimensionless work output coefficient as follows:

$$C_{\rm tot} = C_{\rm prop} + C_c = \frac{\eta_{\rm prop}\dot{W}_{\rm prop}}{\dot{m}_0 Cp_0 T_0} + \frac{F_c V_0}{\dot{m}_0 Cp_0 T_0} \tag{1}$$

The equivalent shaft horsepower (ESHP) [11], which takes into account the produced shaft horse power (SHP) and the residual thrust, in flight and at takeoff static condition is given by

$$\text{ESHP} = \text{SHP} + \frac{FV_0}{\eta_{\rm prop}} \tag{2}$$

and

$$\text{ESHP} = \text{SHP} + \frac{F}{2.5}$$

respectively. The specific power and PSFC

$$W_s = C_{\rm tot} Cp_0 T_0 \tag{3}$$

and

$$\text{PSFC} = \frac{f}{C_{\rm tot} Cp_0 T_0}$$

The fuel-to-air ratio is obtained iteratively from the enthalpies of an air and gas mixture:

$$f = \frac{h_{tg}(T_{t4}, f) - h_{t3}(T_{t3})}{\eta_b Lcv - h_{tg}(T_{t4}, f)} \tag{4}$$

and

$$h_{tg} = \frac{h_{ta} + f h_{tgp}}{1 + f}$$

Considering the values of efficiencies and loss coefficients of engine components [12], depending on the technology level, air cooling fractions [13], and mixing losses, the work delivered to the propeller is given for the three variants as follows:

$$\frac{\dot{W}_{\rm prop}}{\dot{m}_0 Cp_0 T_0} = \eta_g \eta_m (1+f) \frac{Cp_{4,5}}{Cp_4} \tau_b \tau_{m_T} (1 - \tau_T) - \frac{Cp_{2,3}}{Cp_0} \eta_g \tau_r (\tau_C - 1) \tag{5a}$$

$$\frac{\dot{W}_{\rm prop}}{\dot{m}_0 Cp_0 T_0} = \eta_g \eta_{mL} (1+f) \frac{Cp_{45,5}}{Cp_4} \tau_b \tau_{m_{TH}} \tau_{m_{TL}} \tau_{TH} (1 - \tau_{TL}) \tag{5b}$$

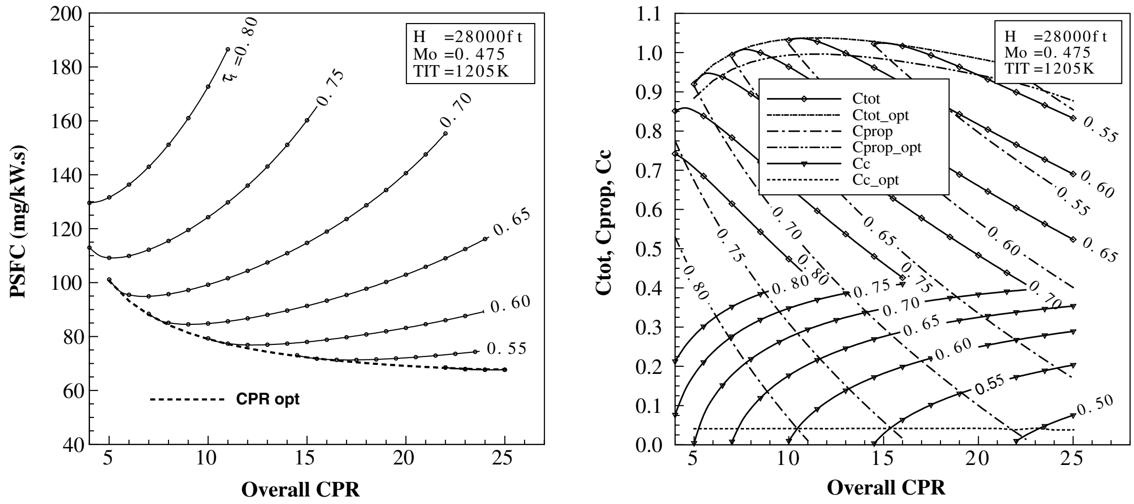
$$\frac{\dot{W}_{\rm prop}}{\dot{m}_0 Cp_0 T_0} = \eta_g \eta_{mL} (1+f) \frac{Cp_{45,5}}{Cp_4} \tau_b \tau_{m_{TH}} \tau_{m_{TL}} \tau_{TH} (1 - \tau_{TL}) - \frac{Cp_{2,23}}{Cp_0} \eta_g \tau_r (\tau_{CL} - 1) \tag{5c}$$

A preliminary parametric study in on-design (cruise flight) allowed the procurement of the evolutions of specific power, optimum turbine temperature ratio, PSFC, and work output coefficients. Figure 2, plotted for a single-spool fixed turbine, shows less power produced at a high turbine expansion temperature ratio (TTR), but contrarily at low TTR, more power is recovered with less fuel consumption. The core work output coefficient tends to increase with CPR, but it remains almost constant for an optimum TTR.

CPR in cruise for the maximum total specific power (dashed line in Fig. 2) corresponds approximately to a value of 11, which is near the reference engine design. The space of design highlights two distinct design points for the minimum fuel consumption and maximum specific power, which needs tradeoffs with regard to engine technology.

B. Offdesign Performance

Offdesign parameters at takeoff are estimated iteratively based on the conservation of work and mass flow, assuming choked flow conditions at turbine nozzle guide vanes, but the exhaust nozzle is mainly unchoked:



a) b)

Fig. 2 Single-spool fixed turbine plots for a) PSFC and b) work output coefficients.

Variant 1:

$$\tau_C = 1 + k_1 \frac{1-\tau_T}{\tau_r}\frac{T_{t4}}{T_0} - \frac{k_2}{\tau_r}\left[\frac{\dot{W}_{\text{prop}}}{\dot{m}_0 C p_0 T_0}\right] \tag{6a}$$

$$\pi_T = k_3\sqrt{\tau_T}/\text{MFP}_9 \tag{6b}$$

Variant 2:

$$\tau_C = 1 + k_4 \frac{T_{t4}/T_0}{\tau_r} \tag{7a}$$

$$\pi_{TL} = k_5\sqrt{\tau_{TL}}/\text{MFP}_9 \tag{7b}$$

Variant 3:

$$\tau_{CH} = 1 + k_8 \frac{T_{t4}/T_0}{\tau_r \tau_{CL}} \tag{8a}$$

$$\tau_{CL} = 1 + k_6 \frac{(1-\tau_{TL})}{\tau_r}\frac{T_{t4}}{T_0} - \frac{k_7}{\tau_r}\left[\frac{\dot{W}_{\text{prop}}}{\dot{m}_0 C p_0 T_0}\right] \tag{8b}$$

$$\pi_{TL} = k_9\sqrt{\tau_{TL}}/\text{MFP}_9 \tag{8c}$$

Updated mass flow follows this relation:

$$\dot{m}_0 = k_{10} p_0 \pi_r \pi_C/\sqrt{T_{t4}} \tag{9}$$

The validation results based on the reference engine Allison 501-D22A [8] are in good agreement, both in cruise and takeoff operations.

IV. Engine Cycle Parameters Optimization

Higher TITs are desirable but require expensive alloys and cooling techniques, on the other side, the benefits from increasing CPR must be reconsidered for weight and complexity reasons. Physical constraints corresponding to limitations in design variables and correct operating conditions should be considered. For example, the compressor exit temperature and turbine entry temperature should not exceed certain values.

A. Statement of Multi-Objective Real Design Problems Optimization

This MOP consists of finding the decision variables, optimizing two objectives simultaneously, and satisfying constraints. The PSFC has to be minimized and the specific power W_s has to be maximized. For the case of a fixed turbine turboprop (for example) to be optimized in cruising flight, the sets of design parameters have to be found by satisfying the following:

$$\text{Obj} = \min[\text{PSFC}, 1/W_s] \tag{10}$$

Subject to

$$[ESHP]_{TO} \geq 4795 \text{ hp}, \qquad [\text{T}_{t3}]_{TO} \leq 750 \text{ K}$$
$$1100 \text{ K} \leq \text{TIT} \leq 1600 \text{ K}, \qquad 0.4 \leq \tau_T \leq 0.8$$
$$7 \leq \pi_C \leq 25$$

To deal with this optimization, traditional methods based on transforming the MOP into a single objective through an aggregating approach (weighted sum, goal programming, etc.) can be used. Weighted sum formulation converts this problem into a scalar one by constructing a weighted sum of individual objectives. Because of the conflicting nature of the objectives, there is no unique solution, but rather a set of solutions that can be classified into dominated and nondominated solutions representing the best compromise distributed on the so-called Pareto front (Fig. 3). The evaluation function is the weighted sum of two objectives written in terms of normalized values, because they have different units and magnitudes, for which the weight factors ϖ are changing evenly from 0 to 1:

$$\text{Obj} = \varpi\frac{\text{PSFC} - (\text{PSFC})_I}{(\text{PSFC})_M - (\text{PSFC})_I} - (1-\varpi)\frac{W_s - (W_s)_M}{(W_s)_I - (W_s)_M} \tag{11}$$

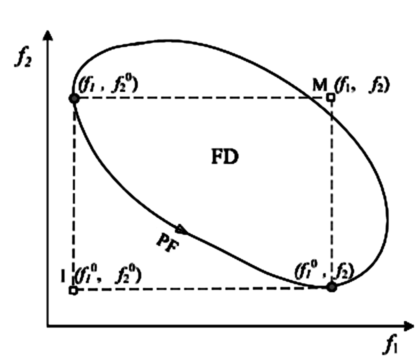


Fig. 3 Pareto frontier (FD denotes feasible domain and PF denotes Pareto front).

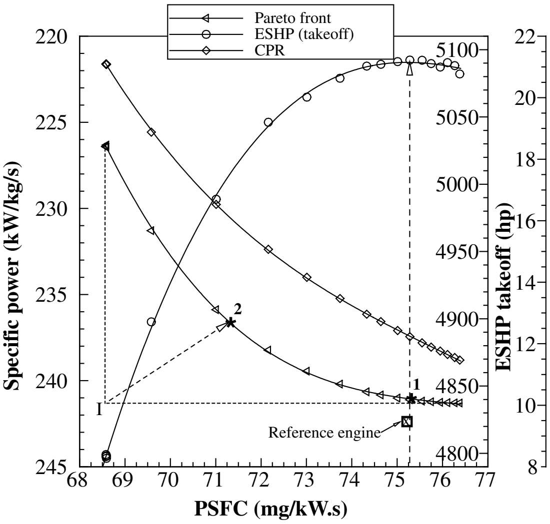


Fig. 4 Design choice for a single-shaft turboprop engine, upper limit TIT = 1390 K.

Index I stands for the ideal point $[(\text{PSFC})_I$ and $(W_s)_I]$ resulting from separate minimization of PSFC and maximization of the specific power, whereas, subscript M stands for an opposite point $[(\text{PSFC})_M$ and $(W_s)_M]$.

B. Constraints Handling

The central problem for applying a GA is how to handle constraints. First versions of GAs employed a do-not-go strategy, which later was superseded by the penalty function, for which constraints were turned into adjustments to the measure of merit applied to degrade the calculated value of the objective function. Recently, Deb [14] developed a constraint handling method (used in present algorithm) based on a penalty function that could provide distinction between feasible and infeasible solutions in terms of constraint violation. Mathematically, the fitness function can be written in the following form:

$$\text{Obj} = \begin{cases} f & \text{if } g_j \geq 0 \\ f_{\max} + \sum_{j=1}^{Nc} |g_j| & \text{otherwise} \end{cases} \tag{12}$$

C. Subroutine PIKAIA

PIKAIA is a general purpose GA that seeks to maximize a user-defined function in a bounded n-dimensional space [15]. The restriction of a normalized parameter allows greater flexibility and portability across problem domains. The maximization is carried out on a population made up of individuals (trial solutions) that remains fixed throughout the evolution. Rather than evolving the population until some tolerance criterion is satisfied, this subroutine carries the evolution over a user-defined preset number of generations. A main calling program and external routines are provided to evaluate the objectives and constraint functions that communicate with the engine performance analyzer.

V. Results and Discussion

Optimization of the propulsion cycle parameters for three variants of turboprops is based on the power requirements in cruise (on-design) and takeoff (offdesign), in addition to technology constraints. Limitations for compressor exit temperature and power turbine entry temperature are shown to be most effective at takeoff.

The configuration of a single-spool fixed turbine turboprop was optimized for an upper limit of TIT of 1390 K (as in the reference engine model) and 1550 K (fourth-generation technology).

Figure 4 shows the feasible domain and the tradeoff surface of the Pareto front obtained by biobjective optimization for a single shaft fixed turbine turboprop. At the same cruising flight condition, the reference engine data [8] give an ESHP of 2200 hp (1640.54 kW) and a PSFC of 0.445 lb/h/hp (75.20 mg/kW · s). The obtained power range corresponding to the Pareto front is above the minimum requested power at takeoff, and the design point of the reference engine is shown to be close to the point of maximum power. According to the reference design point (in square) for this STOL aircraft, the designer has given advantage to the power at takeoff vis-à-vis PSFC. Figure 4 shows that points 1 and 2 on the Pareto front may constitute two design choices for this configuration of a turboprop. As presented in Table 1, when considering the criterion of maximum ESHP at takeoff, point 1 constitutes a good choice. On the other side, if we intend to improve the fuel consumption, point 2 near the ideal point I may be selected, allowing a benefit in PSFC (about 5.2%) despite a loss in ESHP (about 1.6%), but for an increase (31.6%) in CPR. For the relatively low CPR values, the optimization is basically guided by the minimum power at takeoff (Fig. 5).

For the maximum allowed TIT of 1550 K (fourth-generation, cooled first-stage turbine), when considering the criterion of maximum power at takeoff, point 1 is a good choice. On the other side, point 2 gives a 3.6% gain in PSFC for a small loss in ESHP at takeoff but for an increase in CPR (20.9%). When compared with the third generation, there is a gain in PSFC (8–6.4%), and ESHP at takeoff is increased by 29.6–31.1% [but at an increase in CPR (25.6–36.8%)]. To keep the same required power, we may reduce the size and weight of the engine. Figures 6a and 6b show convergence history of TIT and CPR given for weight factors evenly distributed from 0 to 1. It can be observed that, in early generations, these design parameters oscillated but became well directed toward the target values. Convergence of TIT required a number of generations (around 80), and all curves converged to the same value. Convergence of CPR required a number of generations (around 80–100) but converged values depended on weight factors. The best population size was related to the length of the chromosome and problem complexity. According to our computation experience, the length of the chromosome and the population size had values of 6 and 100.

For the second variant with free power turbine, two proposed design points are suggested. The first one corresponds to a maximum ESHP at takeoff (point 1) and near the ideal point 2 (Table 1) that

Table 1 Suggested design points for three variants of turboprops

	Single-spool fixed turbine				Single-spool free turbine				Twin-spool fixed turbine			
	Limit TIT = 1390 K		Limit TIT = 1550 K		Limit TIT = 1390 K		Limit TIT = 1550 K		Limit TIT = 1390 K		Limit TIT = 1550 K	
	1	2	1	2	1	2	1	2	1	2	1	2
Overall CPR	12.25	16.13	16.76	20.27	13.19	16.76	15.67	19.01	11.85	15.31	17.56	22.33
HP CPR	——	——	——	——	——	——	——	——	8.72	9.50	15.18	19.43
TIT, K	1203.7	1203.7	1342.3	1342.3	1203.7	1203.7	1342.3	1342.3	1203.7	1203.7	1342.3	1342.3
Power TTR	0.5892	0.5538	0.5569	0.5328	0.7401	0.7312	0.7165	0.7076	0.7241	0.7015	0.6958	0.6833
PSFC, mg/kW · s	75.23	71.28	69.21	66.71	73.01	69.75	69.02	66.53	73.66	69.61	66.60	63.54
W_s , kW/kg/s	241.05	236.53	307.63	305.04	243.81	238.98	308.71	306.05	248.29	245.68	316.22	312.54
ESHP,hp	5090.5	5006.6	6599.3	6565.6	5011.0	4928.2	6473.2	6441.7	5009.9	4855.2	6641.8	6602.8

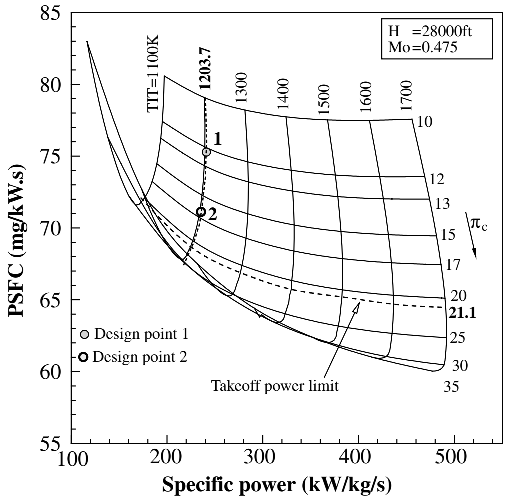
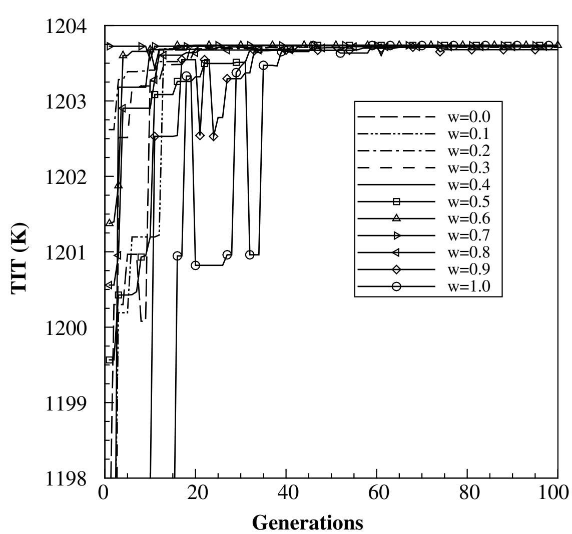
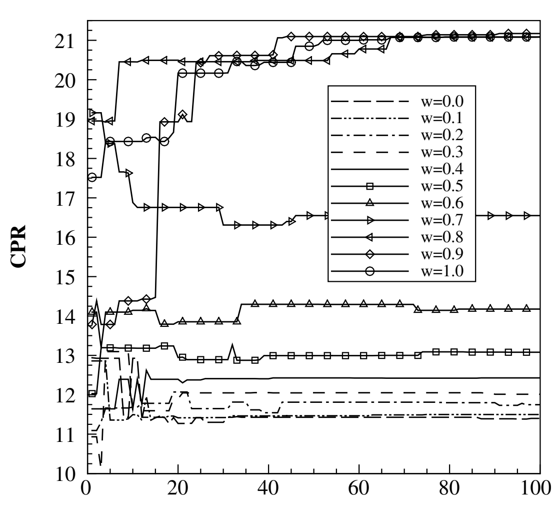


Fig. 5 Design space for a single-shaft turboprop engine, upper limit TIT = 1390 K.



a) Convergence of TIT



b) Convergence of CPR

Fig. 6 Convergence histograms for a single-spool fixed turbine, upper limit TIT = 1390 K.

allowed a benefit in PSFC (about 4.4%) despite a small loss in power (about 1.6%) and an increase (27%) in CPR. Optimization presented for a fourth generation of a turboprop by cooling power turbine led to an extended Pareto front in comparison to an uncooled turbine, and there was a reduction in CPR, which was an advantage in addition to extending the life of the blades.

For the last variant of twin-spool with fixed turbine characterized by one more variable, the high pressure compression ratio, the two proposed design points (Table 1) corresponding to a maximum ESHP at takeoff (point 1), and the point 2 near the ideal point allowed a benefit in PSFC of about 5.5%, but it was at an expense (29.2%) of the CPR. For the upper limit TIT of 1550 K [fourth-generation cooled lower pressure ratio (LP) turbine], two design points were proposed: point 1 corresponded to maximum power at takeoff and point 2 resulted in a lower PSFC (about 4.6%), but it was at an expense (27.1%) in CPR. By selecting a high TIT for this configuration, the value of ESHP at takeoff is increased (32.5–36%) and PSFC reduced (8.5–8.7%), but the overall CPR is increased (48–45.8%). Nevertheless, we can reduce engine size to keep the minimum requested power. By comparing the three optimized designs based on criterions 1 and 2, the last configuration seemed to give the better performance. The decision to design either according to criterions 1 or 2 will depend on tradeoffs between engine weight and flight range. From the point of view of design for the twin-spool fixed turbine, when operating at the design TITs (1203 and 1342 K), we may use two stages of LP compressor and two HP centrifugal compressors, or several stages of HP axial compressor.

VI. Conclusions

Optimization of design parameters of three configurations of turboprops matching the power requirements of a propeller-driven aircraft is presented in this paper. Two design options arise from the Pareto-front solutions: the first is a compromise near the ideal point and the second is that of maximum takeoff power. Computations show the superiority of such a GA algorithm in rapidly optimizing and preserving the diversity of nondominated individuals, as well as the quality of Pareto fronts. The present methodology may be used in the preliminary design because of its ease in selecting and optimizing the design of a turboprop. This latter is further being improved to include other design criteria, such as engine size and weight, as well as a hybrid algorithm that combines the GA with a gradient-based algorithm to increase the globality of optimization exploration.

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